Pattern formation in *Escherichia coli*: A model for the pole-to-pole oscillations of Min proteins and the localization of the division site

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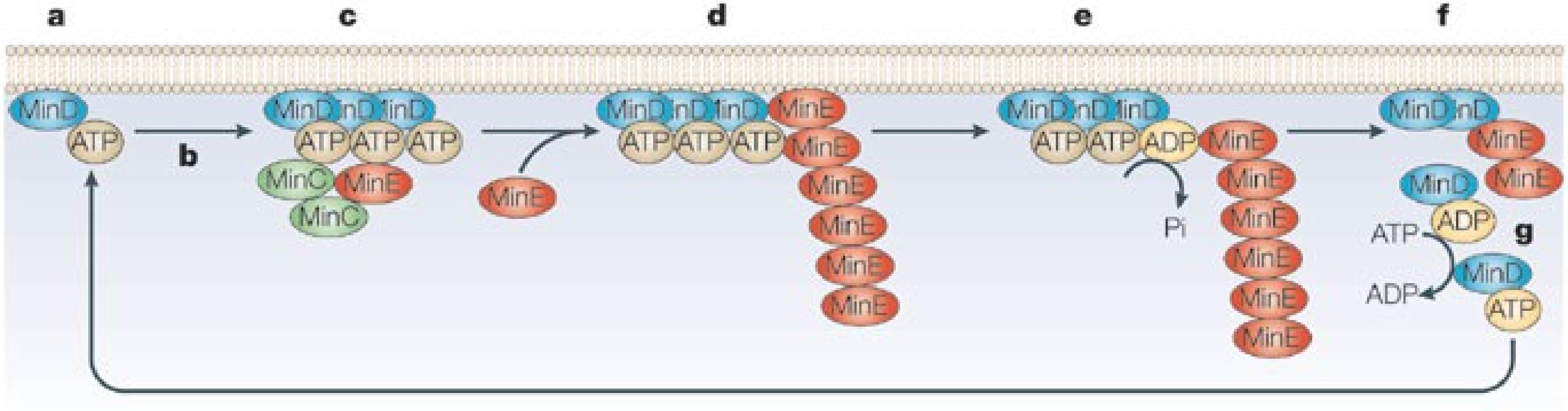
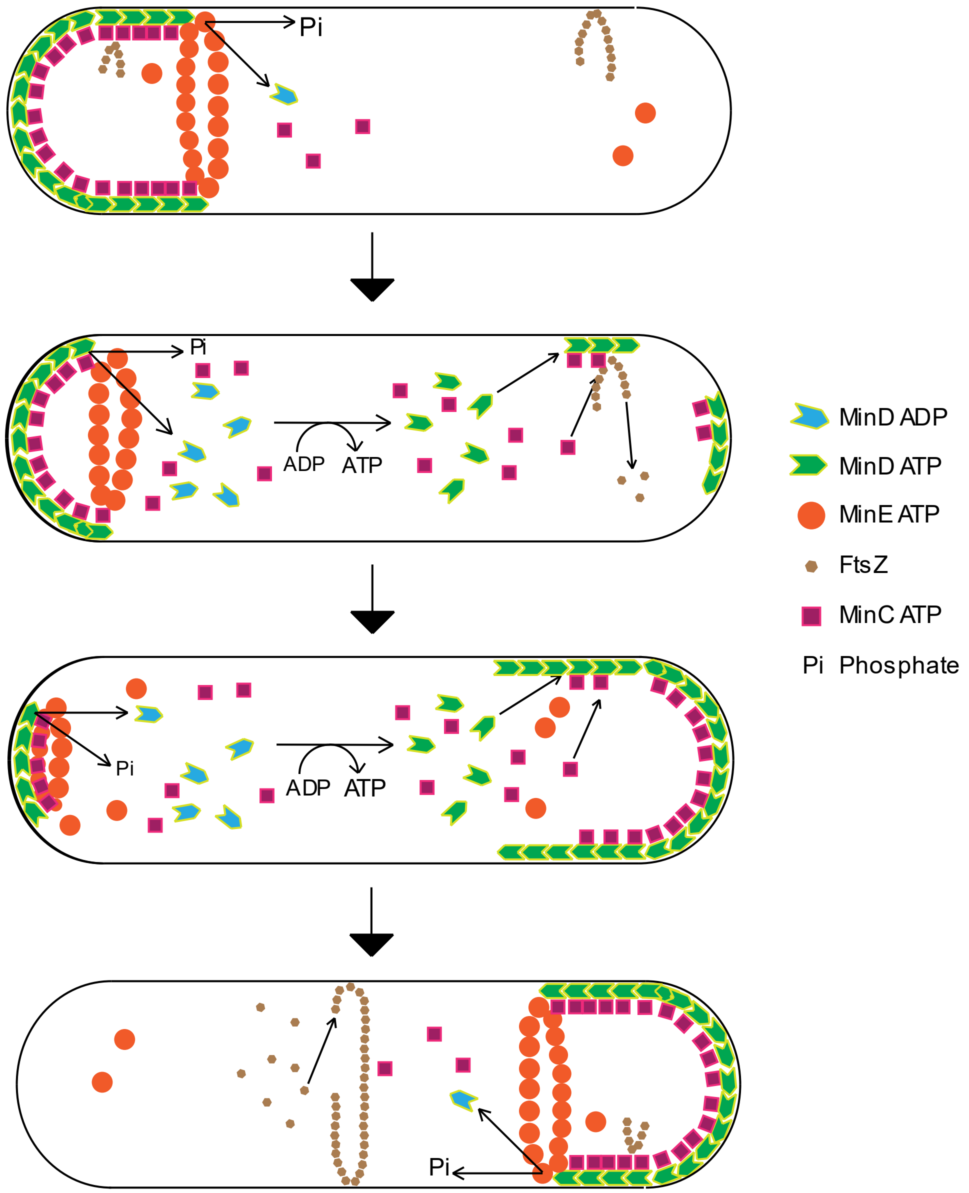
4 Proteins involved:
MinC, MinD, MinE
And Ftsz, which forms the ring.



Translate Biology to Chemistry and to Physics



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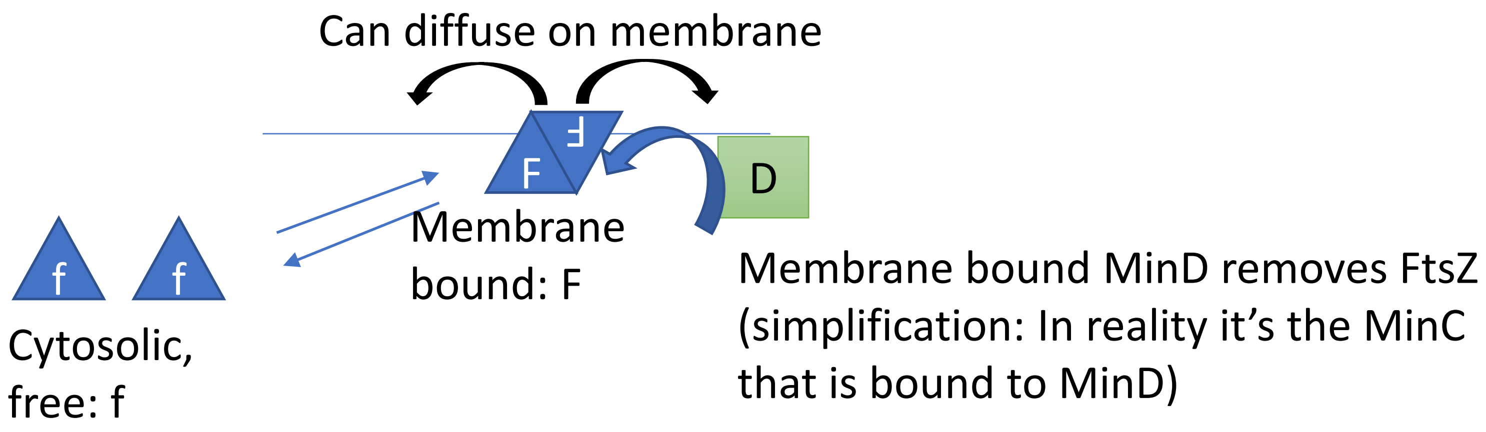
1) We only look at FtsZ



$$\frac{\partial F}{\partial t} = \rho_{\mathrm{F}} f$$

$$\frac{\partial f}{\partial t} = \sigma_{\mathrm{f}} - \rho_{\mathrm{F}} f \frac{F^2 + \sigma_{\mathrm{F}}}{1 + \kappa_{\mathrm{F}} F^2} - \mu_{\mathrm{f}} f + \mathcal{D}_{\mathrm{f}} \frac{\partial^2 f}{\partial x^2}$$

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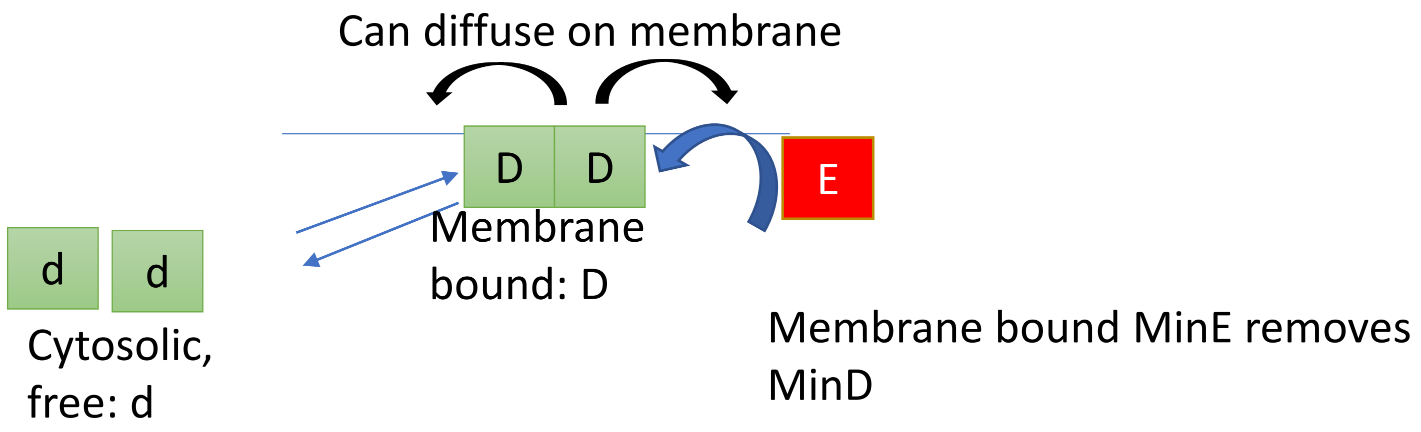
2) We now look at MinD. For simplicity we take MinD and MinC together



$$\frac{\partial D}{\partial t} = \rho_{\mathrm{D}} d$$

$$\frac{\partial d}{\partial t} = \sigma_{\mathrm{d}} - \rho_{\mathrm{D}} d(D^2 + \sigma_{\mathrm{D}}) - \mu_{\mathrm{d}} d + \mathcal{D}_{\mathrm{d}} \frac{\partial^2 d}{\partial x^2}$$

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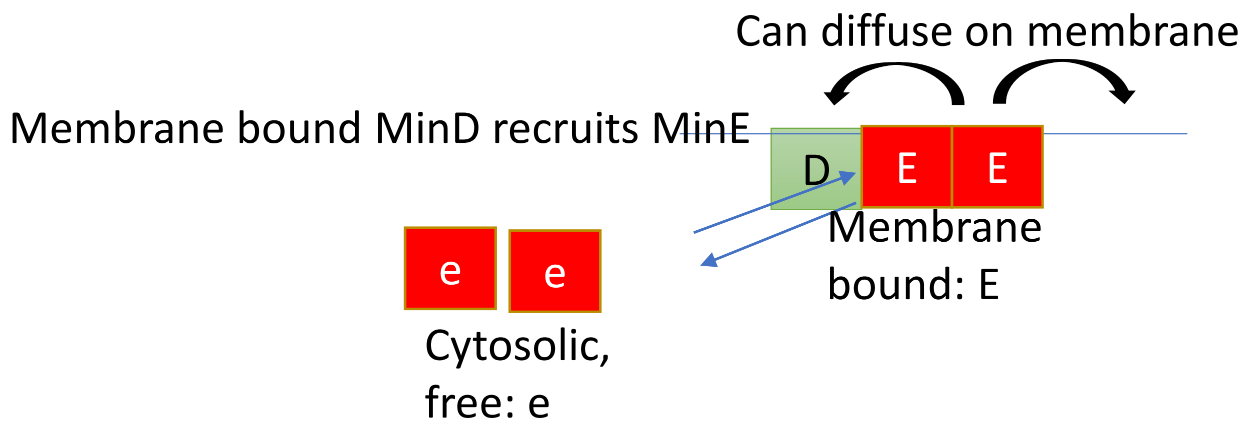
3) Finally we look at MinE.



$$\frac{\partial E}{\partial t} = \rho_{\mathrm{E}} e$$

$$\frac{\partial e}{\partial t} = \sigma_{\mathrm{e}} - \rho_{\mathrm{E}} e \frac{D}{(1 + \kappa_{\mathrm{DE}} D^2)} \frac{(E^2 + \sigma_{\mathrm{E}})}{(1 + \kappa_{\mathrm{E}} E^2)} - \mu_{\mathrm{e}} e + \mathcal{D}_{\mathrm{e}} \frac{\partial^2 e}{\partial \mathrm{x}^2}$$

And here the full model ☺

$$\frac{\partial F}{\partial t} = \rho_{\mathrm{F}} f \frac{F^2 + \sigma_{\mathrm{F}}}{1 + \kappa_{\mathrm{F}} F^2} - \mu_{\mathrm{F}} F - \mu_{\mathrm{DF}} DF + \mathcal{D}_{\mathrm{F}} \frac{\partial^2 F}{\partial \mathrm{x}^2}$$

$$\frac{\partial f}{\partial t} = \sigma_{\mathrm{f}} - \rho_{\mathrm{F}} f \frac{F^2 + \sigma_{\mathrm{F}}}{1 + \kappa_{\mathrm{F}} F^2} - \mu_{\mathrm{f}} f + \mathcal{D}_{\mathrm{f}} \frac{\partial^2 f}{\partial \mathrm{x}^2}$$

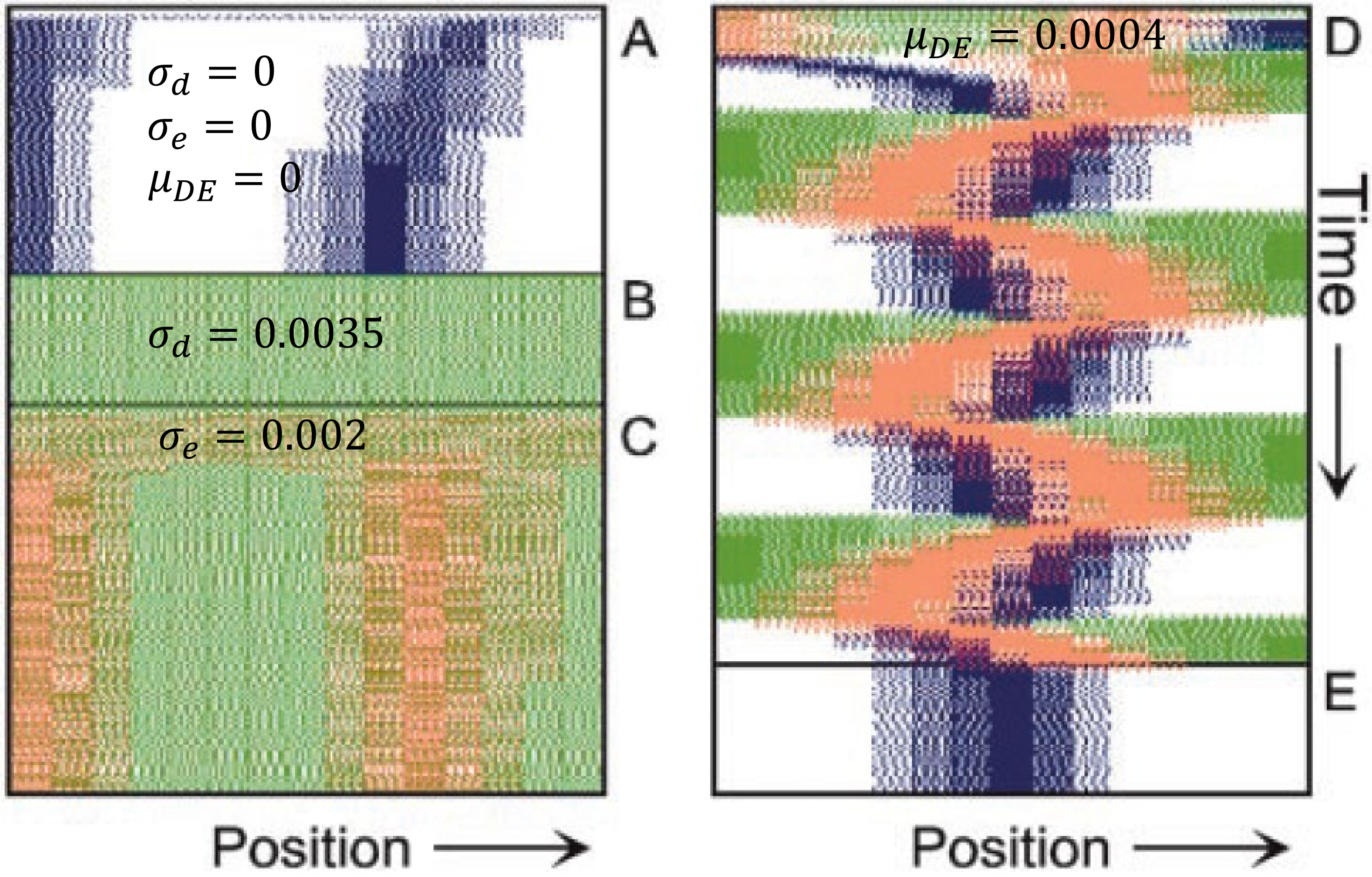
$$\frac{\partial D}{\partial t} = \rho_{\mathrm{D}} d(D^2 + \sigma_{\mathrm{D}}) - \mu_{\mathrm{D}} D - \mu_{\mathrm{DE}} DE + \mathcal{D}_{\mathrm{D}} \frac{\partial^2 D}{\partial \mathrm{x}^2}$$

$$\frac{\partial d}{\partial t} = \sigma_{\mathrm{d}} - \rho_{\mathrm{D}} d(D^2 + \sigma_{\mathrm{D}}) - \mu_{\mathrm{d}} d + \mathcal{D}_{\mathrm{d}} \frac{\partial^2 d}{\partial \mathrm{x}^2}$$

$$\frac{\partial E}{\partial t} = \rho_{\mathrm{E}} e \frac{D}{(1 + \kappa_{\mathrm{DE}} D^2)} \frac{(E^2 + \sigma_E)}{(1 + \kappa_{\mathrm{E}} E^2)} - \mu_{\mathrm{E}} E + \mathcal{D}_{\mathrm{E}} \frac{\partial^2 E}{\partial \mathrm{x}^2}$$

$$\frac{\partial e}{\partial t} = \sigma_{\mathrm{e}} - \rho_{\mathrm{E}} e \frac{D}{(1 + \kappa_{\mathrm{DE}} D^2)} \frac{(E^2 + \sigma_{\mathrm{E}})}{(1 + \kappa_{\mathrm{E}} E^2)} - \mu_{\mathrm{e}} e + \mathcal{D}_{\mathrm{e}} \frac{\partial^2 e}{\partial \mathrm{x}^2}$$

Results as in the paper:



$$\frac{\partial F}{\partial t} = \rho_{\mathrm{F}} f \frac{F^2 + \sigma_{\mathrm{F}}}{1 + \kappa_{\mathrm{F}} F^2} - \mu_{\mathrm{F}} F - \mu_{\mathrm{DF}} DF + \mathcal{D}_{\mathrm{F}} \frac{\partial^2 F}{\partial x^2}$$

$$\frac{\partial f}{\partial t} = \sigma_{\mathrm{f}} - \rho_{\mathrm{F}} f \frac{F^2 + \sigma_{\mathrm{F}}}{1 + \kappa_{\mathrm{F}} F^2} - \mu_{\mathrm{f}} f + \mathcal{D}_{\mathrm{f}} \frac{\partial^2 f}{\partial x^2}$$

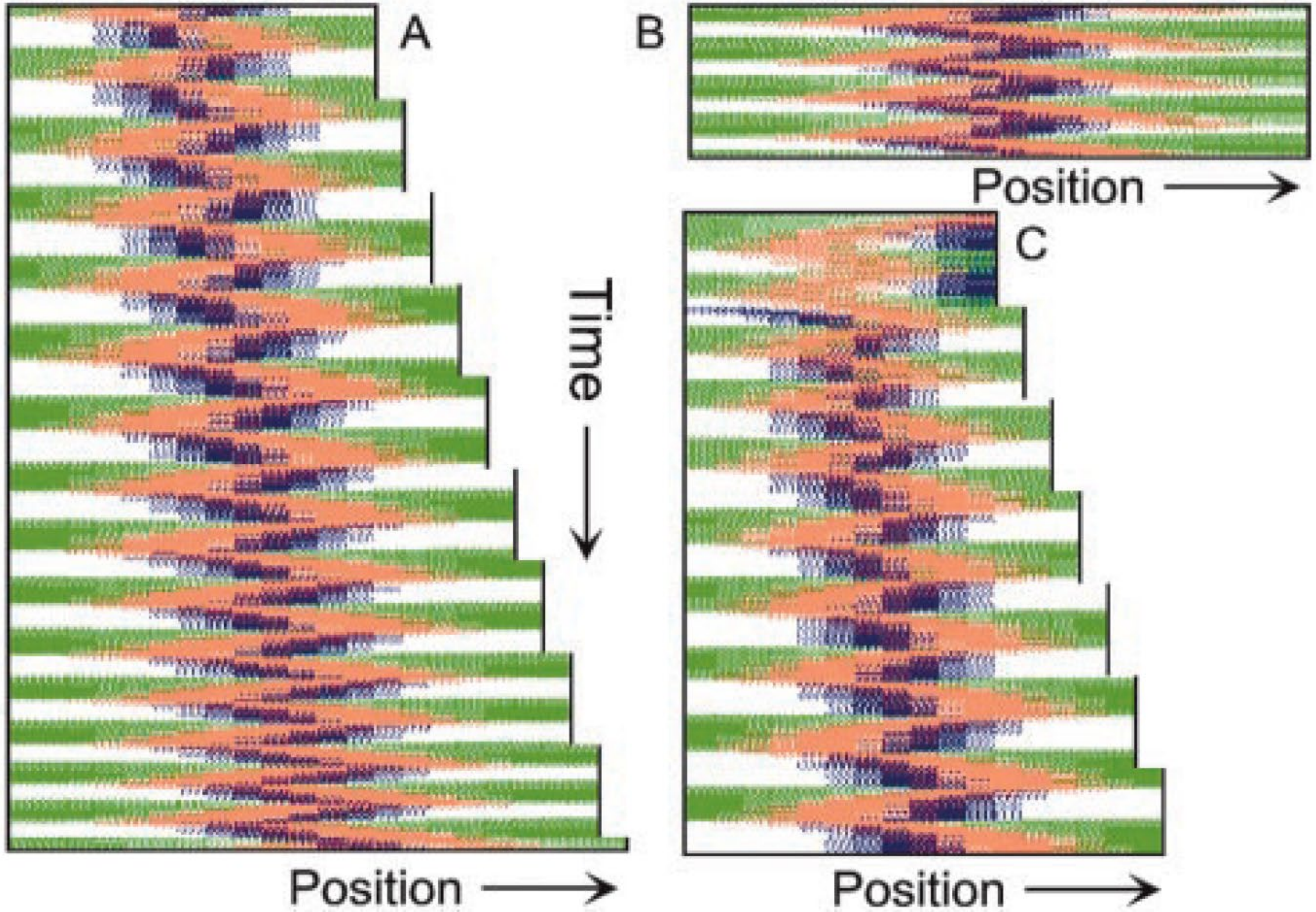
$$\frac{\partial D}{\partial t} = \rho_{\mathrm{D}} d (D^2 + \sigma_{\mathrm{D}}) - \mu_{\mathrm{D}} D - \mu_{\mathrm{DE}} DE + \mathcal{D}_{\mathrm{D}} \frac{\partial^2 D}{\partial x^2}$$

$$\frac{\partial d}{\partial t} = \sigma_{\mathrm{d}} - \rho_{\mathrm{D}} d (D^2 + \sigma_{\mathrm{D}}) - \mu_{\mathrm{d}} d + \mathcal{D}_{\mathrm{d}} \frac{\partial^2 d}{\partial x^2}$$

$$\frac{\partial E}{\partial t} = \rho_{\mathrm{E}} e \frac{D}{(1 + \kappa_{\mathrm{DE}} D^2)} \frac{(E^2 + \sigma_{E})}{(1 + \kappa_{\mathrm{E}} E^2)} - \mu_{\mathrm{E}} E + \mathcal{D}_{\mathrm{E}} \frac{\partial^2 E}{\partial x^2}$$

$$\frac{\partial e}{\partial t} = \sigma_{\mathrm{e}} - \rho_{\mathrm{E}} e \frac{D}{(1 + \kappa_{\mathrm{DE}} D^2)} \frac{(E^2 + \sigma_{\mathrm{E}})}{(1 + \kappa_{\mathrm{E}} E^2)} - \mu_{\mathrm{e}} e + \mathcal{D}_{\mathrm{e}} \frac{\partial^2 e}{\partial x^2}$$

Growth and division



Problems if no division